

# Properties of Nuclear Matter in A Nonlinear Realized Approach of the SU(2) Chiral Symmetry Spontaneous Breaking

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## Abstract

A nonlinear realization of SU(2) chiral symmetry spontaneous breaking approach is developed in the composite operator formalism. The properties of nuclear matter, the pion and the quark condensates in nuclear matter are evaluated. The calculated results show that the saturation properties of nuclear matter can be reproduced well with the approach. Meanwhile, the expectation value of  $\bar{\pi}^2$  becomes nonzero in the nuclear matter with minimal density  $\rho_N \sim 0.235\text{fm}^{-3}$ , and the quark condensate decreases monotonously with the increasing of the nuclear matter density. As the pion condensate appears, the decreasing rate of the quark condensate is enhanced.

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It is known that Quantum chromodynamics (QCD) has a non-trivial vacuum with non-perturbative condensates of quarks and gluons. In the low energy region, QCD has two very important properties: chiral symmetry spontaneous breaking and confinement. It is believed that the two properties are closely related to the vacuum characteristics of QCD. The properties of the QCD vacuum have been investigated with various approaches[1, 2]. Based on the vacuum structure, some hadron properties have been described well [1, 3, 4, 5, 6]. The investigations show that the chiral symmetry spontaneous breaking plays an important role in understanding the feature of the low energy strongly interacting physics[7]. From the Goldstone's theorem, Goldstone bosons appear as the chiral symmetry is spontaneously broken. To carry out the constraint by appearance of Goldstone bosons, several realization formalisms of the chiral symmetry spontaneous breaking have been developed [8, 9]. However, linear approximation should be made in practical calculation within the above mentioned schemes. In the spirit of these formalisms and the composite operator scheme[10] of the QCD, we propose an approach which realizes the chiral symmetry spontaneous breaking nonlinearly not only in formalism but also in practical calculation.

On the other hand, chiral symmetry may be restored gradually in nuclear matter as the increasing of its density. As a consequence, the quark condensates in nuclear matter decrease gradually with the increasing of nuclear matter density. The quark condensates in finite density can be evaluated with QCD sum rules approach[11]. With the Hellmann-Feynman theorem[12] being implemented, one can also evaluate the quark condensates at hadronic level. Almost all the approaches can give the descent feature in nuclear matter in the low density region. However an upturn emerges at higher density in the linear Walecka model[13], Dirac-Brueckner method[14], Dyson-Schwinger formalism[15]. Even though the upturn can be eliminated [16] with the Brown-Rho scaling[17] being included in the Walecka model, the quark condensate vanishes at a density  $\rho \approx 3.5\rho_0$ . Such models do not show the connection between the chiral symmetry spontaneous breaking and the quark condensate. With the Hellmann-Feynman theorem being included in the present formalism, we evaluate the quark condensate too.

In general, symmetry means the invariance under a certain transformation. For the linear infinitesimal transformation

$$\phi'_n(x) = \phi_n + i\epsilon \sum_m t_{nm}^\alpha \phi_m(x) \quad (1)$$

where  $t^\alpha$  is a generator of the symmetry group of the Lagrangian of the system and  $\phi_n$  is spin-zero boson field or spin-zero composite operators. In the special case of constant fields, the quantum effective potential  $V(\phi)$  can be introduced which has the symmetry property

$$\sum_{n,m} \frac{\partial V(\phi)}{\partial \phi_n} t_{nm}^\alpha \phi_m = 0 \quad (2)$$

where  $\phi$  is independent of  $x$ . The symmetry spontaneous breaking appears when  $\frac{\partial V(\bar{\phi})}{\partial \phi_m} = 0$

and  $\bar{\phi}_m \neq 0$ . It also means that

$$\sum_{n,m} \frac{\partial^2 V(\phi)}{\partial \phi_n \partial \phi_l} t_{nm}^\alpha \bar{\phi}_m = 0. \quad (3)$$

Thus, if the symmetry is broken,  $\sum_m t_{nm}^\alpha \bar{\phi}_m$  is not identically equal to zero with  $\bar{\phi}_m = \langle 0 | \phi_m | 0 \rangle$ . The massless eigenvectors of the mass matrix  $\Delta_{nl}^{-1}(0) = \frac{\partial^2 V(\phi)}{\partial \phi_n \partial \phi_l}$  span a linear space formed by independent linear combination of the vectors. We note that the Goldstone bosons lie in this linear space and remainder has to be perpendicular to the space. This constraint leads to the nonlinear realization of chiral symmetry spontaneous breaking of  $SU_L(2) \times SU_R(2)$ . We formulate then the nonlinear realization of  $SU_L(2) \times SU_R(2)$  on the quark level at first.

The fundamental ingredients in the composite operator scheme are the following four composite operators

$$\psi_4 = \bar{q}q, \quad \psi_i = i\bar{q}\tau_5\tau_i q \quad (i = 1, 2, 3), \quad (4)$$

where  $q$  is the  $\{u, d\}$  quark fields and  $\tau_i (i = 1, 2, 3)$  are the Pauli matrices. The transformation of the  $\psi_\alpha (\alpha = 1, 2, 3, 4)$  under the chiral symmetry transformation of the quarks  $e^{ir_5\epsilon^i\tau_i}$  can be given as

$$\delta\psi_4 = 2\epsilon^i\psi_i, \quad \delta\psi_i = -2\epsilon^i\psi_4 \quad (5)$$

where  $\epsilon^i$  are the infinitesimal parameters.

If the vacuum expectation value of quarks is not zero, i.e.,  $\langle 0 | \bar{q}(x)q(x) | 0 \rangle \neq 0$ , the chiral symmetry  $SU_L(2) \times SU_R(2)$  is spontaneously broken. To realize the constraint on the breaking (shown by Eq.(3)), the  $\psi_\alpha$  have to be separated into two parts: one contains the Goldstone bosons and the other does not. Therefore, we write

$$\psi_\alpha = \sum_{\beta} (e^{2i\xi^i T^{i4}})_{\alpha\beta} \tilde{\psi}_\beta \quad (6)$$

where  $T^{\alpha\beta}$  are the generators of the four dimensional rotation group, for instance,

$$iT^{14} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad iT^{24} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad iT^{34} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix},$$

The condition that  $\tilde{\psi}_\alpha$  does not contain Goldstone modes is then formulated as

$$\sum_{\alpha,\beta} \tilde{\psi}_\alpha(x) [T^{\mu\nu}]_{\alpha\beta} \langle 0 | \psi_\beta | 0 \rangle = 0. \quad (7)$$

It is easy to find that  $\tilde{\psi}_\alpha$  can be taken as  $\tilde{\psi}_4$  and other three components are zero. In the infinitesimal form

$$\delta\psi_i = 2\xi^i \tilde{\psi}_4 \quad (8)$$

Comparing the Eq.(8) with the Eq.(5), we obtain

$$q = e^{-ir_5 \xi^i \tau_i} \tilde{q}, \quad (9)$$

where  $\tilde{q}$  is the nonlinear realization of the chiral symmetry.

In order to simplify the factor  $e^{-ir_5 \xi^i \tau_i}$ , we introduce the Goldstone bosons in the form

$$\xi^i = \frac{\tan^{-1}(|\vec{\zeta}| \zeta^i)}{|\vec{\zeta}|}. \quad (10)$$

After a tedious derivation, we obtain the equations to determine the Goldstone bosons  $\zeta^i (i = 1, 2, 3)$  as the following

$$\psi_i = \frac{2\zeta_i}{1 + \vec{\zeta}^2} \tilde{\psi}_4, \quad \psi_4 = \frac{1 - \vec{\zeta}^2}{1 + \vec{\zeta}^2} \tilde{\psi}_4. \quad (11)$$

In the nonlinear realization, the Lagrangian can be constructed only from  $\tilde{q}$ ,  $D_\mu \tilde{q} = \left( \partial_\mu + \frac{2i(\vec{\zeta} \times \partial_\mu \vec{\zeta}) \cdot \vec{t}}{1 + \vec{\zeta}^2} \right) \tilde{q}$  and  $\frac{\partial_\mu \zeta_i}{1 + \vec{\zeta}^2}$ . The Lagrangian density determined by the quantum effective action  $\Gamma$  for the composite operators  $\{\psi_\alpha\}$  can be generally written as

$$\mathcal{L} = -\frac{1}{2A} \partial_\mu \tilde{\psi}_4 \partial^\mu \tilde{\psi}_4 - \frac{2}{A} \tilde{\psi}_4 \tilde{\psi}_4 \vec{D}_\mu \vec{D}^\mu - U(\tilde{\psi}_4), \quad (12)$$

where  $D_\mu^i = \frac{\partial_\mu \zeta^i}{1 + \vec{\zeta}^2}$ .

Since  $\zeta^i$  is a pseudoscalar field and the parity should be conserved, we have  $\langle 0 | \zeta_i | 0 \rangle = 0$ . Then using the assumption of the saturation of vacuum, we can get  $\langle 0 | \psi_4 | 0 \rangle = \langle 0 | \tilde{\psi}_4 | 0 \rangle$ . After a zero-point shift

$$\tilde{\psi}_4 = c + \sigma \sqrt{A}, \quad (13)$$

where  $c$  is a constant, and it equals to the value of quark condensate in vacuum. we can define the  $\sigma$  field. It is remarkable that the  $\sigma$  field induced in this way is not Goldstone boson and it is the remainder for the chiral symmetry spontaneous breaking theory of  $SU(2)_L \times SU(2)_R$ . Meanwhile, the Goldstone bosons and  $\sigma$  appear simultaneously.

In Eq.(12), the second term is the kinetic energy of  $\pi^i$  mesons. The normalization condition requires that  $\frac{2c^2}{A} = \frac{1}{2} f_\pi^2$ . Hence

$$A = 4 \frac{c^2}{f_\pi^2}. \quad (14)$$

The Lagrangian density for  $\sigma$  meson in the existence of current quark mass is

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{g_3}{4} \sigma^4 - m_q \sqrt{A} \sigma \quad (15)$$

In order to destroy the linear term we introduce a new term  $-\frac{1}{2}m_\sigma^2 \left( \sigma + \frac{m_q \sqrt{A}}{m_\sigma^2} \right)^2$ . This means that there is a further shift, then the interaction term  $\frac{1}{4}g_3\sigma^4$  has the following contribution

$$-\frac{3g_3m_q^2A}{2m_\sigma^4} \left( \sigma + \frac{m_q \sqrt{A}}{m_\sigma^2} \right)^2$$

. At the same time, the  $\sigma^3$  term  $\frac{1}{3}g_2\sigma^3$  appears, in which

$$g_2 = -\frac{3m_q \sqrt{A}}{m_\sigma^2} g_3$$

for the quark level and

$$g_2 = -\frac{3\delta M_N \sqrt{A}}{m_\sigma^2} g_3 \quad (16)$$

for the hadronic level, where  $\delta M_N = 50 MeV$ , is assumed as the intrinsic mass of nucleons. In this case the derivative of the mass of the  $\sigma$  meson against that of current quarks is obtained as

$$\frac{dm_\sigma}{dm_q} = \frac{12g_3m_q^2Q^2(0)}{m_\sigma^4f_\pi^2}. \quad (17)$$

From the Gell-Mann–Oakes–Renner relation  $m_\pi^2f_\pi^2 = -4m_qQ(0)$ , it is easy to obtain

$$\frac{dm_\pi}{dm_q} = \frac{m_\pi}{2m_q}. \quad (18)$$

Along the same line as for discussing quarks, we assume that a nucleon has a small intrinsic mass, which characterizes the chiral symmetry explicit breaking at hadronic level, and is referred to as current mass of a nucleon in this paper. This is, in some sense, similar to the current quark mass that induces the explicit chiral symmetry breaking at quark level. The main part of the mass of a nucleon comes from the chiral symmetry spontaneous breaking. Taking the same way as mentioned above and considering the Goldberg-Treiman relation and the partial conservation of the axial current(PCAC), we obtain the Lagrangian density with the nonlinear realization of the SU(2) chiral symmetry spontaneous breaking as

$$\begin{aligned} \mathcal{L}_{ch} = & \bar{\psi} \left( i\gamma_\mu \partial^\mu - M_N - \frac{2\vec{t} \cdot (\vec{\pi} \times \not{\partial} \vec{\pi})}{f_\pi^2 \left( 1 + \frac{\vec{\pi}^2}{f_\pi^2} \right)} + \frac{4iM_N g_A \gamma_5 \vec{t} \cdot \vec{\pi}}{f_\pi \left( 1 + \frac{\vec{\pi}^2}{f_\pi^2} \right)} - \frac{4g_A \gamma_5 \vec{t} \cdot \vec{\pi} \vec{\pi} \cdot \not{\partial} \vec{\pi}}{f_\pi^3 \left( 1 + \frac{\vec{\pi}^2}{f_\pi^2} \right)^2} \right) \psi \\ & - \sigma^2 \bar{\psi} \left( \frac{16ig'M_N \gamma_5 \vec{t} \cdot \vec{\pi}}{f_\pi \left( 1 + \frac{\vec{\pi}^2}{f_\pi^2} \right)} + \frac{16g'\gamma_5 \vec{t} \cdot \vec{\pi} \vec{\pi} \cdot \not{\partial} \vec{\pi}}{f_\pi^3 \left( 1 + \frac{\vec{\pi}^2}{f_\pi^2} \right)^2} \right) \psi - \frac{\delta M_p + \delta M_n}{2} \left( \frac{\left( 1 - \frac{\vec{\pi}^2}{f_\pi^2} \right)}{\left( 1 + \frac{\vec{\pi}^2}{f_\pi^2} \right)} \right) \bar{\psi} \psi \\ & - (\delta M_p - \delta M_n) \left( t_3 - \frac{2}{f_\pi^2} \left( \frac{\pi_3}{1 + \frac{\vec{\pi}^2}{f_\pi^2}} \right) \vec{t} \cdot \vec{\pi} \right) \bar{\psi} \psi - g_\sigma \bar{\psi} \psi \sigma \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - 2\sigma^2 \frac{\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}}{f_\pi^2 \left( 1 + \frac{\vec{\pi}^2}{f_\pi^2} \right)} - \frac{1}{2} m_\pi^2 \vec{\pi}^2, \end{aligned} \quad (19)$$

where  $\vec{t}$  is the isospin matrix vector for isospin  $\frac{1}{2}$  (that is, half the Pauli matrices  $\vec{\tau}$ ).  $M_N$  is the mass of the nucleon caused by the chiral symmetry spontaneous breaking,  $\delta M_p$  and  $\delta M_n$  are the current mass of proton and neutron, respectively. The  $U(\sigma)$  stands for

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4. \quad (20)$$

To represent the repulsive interaction among nucleons and the isospin symmetry breaking in nuclear matter, we include  $\omega$  and  $\rho$  mesons with the following Lagrangians, respectively

$$\mathcal{L}_\omega = -\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu,$$

$$\mathcal{L}_\rho = -\frac{1}{4}\vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu \cdot \vec{\rho}^\mu - g_\rho\bar{\psi}\gamma_\mu\vec{t} \cdot \vec{\rho}^\mu\psi,$$

in which  $\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ ,  $\vec{R}^{\mu\nu} = \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$ .

Since the parity of pion is negative, the expectation value of  $\pi$  is zero in nuclear matter. Neglecting the difference between  $\delta M_p$  and  $\delta M_n$ , i.e., taking  $\delta M_N = \delta M_p = \delta M_n$ , we can write the Lagrangian in the mean-field approximation as

$$\begin{aligned} \mathcal{L}_{RMF} = & \bar{\psi}(i\gamma_\mu\partial^\mu - M_N)\psi - \delta M_N \left( \frac{\left(1 - \frac{\pi_0^2}{f_\pi^2}\right)}{\left(1 + \frac{\pi_0^2}{f_\pi^2}\right)} \right) \bar{\psi}\psi \\ & - g_\sigma\bar{\psi}\psi\sigma_0 - g_\omega\bar{\psi}\gamma^0\psi\omega_0 - g_\rho\bar{\psi}\gamma^0t_3\rho_{03}\psi \\ & - \frac{1}{2}m_\sigma^2\sigma_0^2 - \frac{g_2}{3}\sigma_0^3 - \frac{g_3}{4}\sigma_0^4 - \frac{1}{2}m_\pi^2\pi_0^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2, \end{aligned} \quad (21)$$

where the  $\sigma_0$  is the expectation value of the isoscalar scalar field,  $\pi_0^2$  is that of  $\pi^2$ ,  $\omega_0$  is that of the temporal component for the isoscalar vector field since there is no spatial direction for a uniform nuclear matter at rest,  $\rho_{03}$  is that of  $\rho$  meson in the nuclear matter.

The equations of motion for nucleons and mesons are

$$\left(i\gamma_\mu\partial^\mu - M_N^* - g_\omega\gamma^0\psi\omega_0 - g_\rho\gamma^0\psi t_3\rho_{03}\right)\psi = 0, \quad (22)$$

$$\pi_0^2 = 2\sqrt{\frac{\delta M_N\langle\bar{\psi}\psi\rangle f_\pi^2}{m_\pi^2}} - f_\pi^2, \quad (23)$$

$$m_\sigma^2\sigma_0 + g_2\sigma_0^2 + g_3\sigma_0^3 = -g_\sigma\langle\bar{\psi}\psi\rangle, \quad (24)$$

$$m_\omega^2\omega_0 = g_\omega\langle\bar{\psi}\gamma_0\psi\rangle, \quad (25)$$

$$m_\rho^2\rho_{03} = g_\rho\langle\bar{\psi}\gamma_0t_3\psi\rangle, \quad (26)$$

where  $\langle \bar{\psi}\psi \rangle$ ,  $\langle \bar{\psi}\gamma_0\psi \rangle$  and  $\langle \bar{\psi}\gamma_0 t_3\psi \rangle$  are the scalar density, vector density and isospin density of nuclear matter independently,  $M_N^* = M_N + \delta M_N \left[ \frac{\left(1 - \frac{\pi_0^2}{f_\pi^2}\right)}{\left(1 + \frac{\pi_0^2}{f_\pi^2}\right)} \right] + g_\sigma \sigma_0$  is the effective mass of a nucleon in nuclear matter

Taking the standard process, the energy density and pressure for nuclear matter can be obtained. Then, the properties of nuclear matter can be determined in the chiral Lagrangian Eq. (21).

By fitting the saturation properties of nuclear matter, the parameters with the restriction  $M_N + \delta M_N = 938$  MeV are fixed. Two sets of the parameters are listed in Table 1. For the parameter set  $C_1$  we get the saturation density of  $0.152 \text{ fm}^{-3}$ , binding energy of 15.297 MeV, a compression modulus of 349.10 MeV, symmetry energy coefficient 33.645 MeV and the effective mass of a nucleon of  $0.687 M_N$  for symmetric nuclear matter. The parameter set  $C_2$  corresponds to the saturation density of  $0.151 \text{ fm}^{-3}$ , binding energy of 15.040 MeV, a compression modulus of  $K = 298.88$  MeV, symmetry energy coefficient 32.593 MeV and the effective mass of nucleon of  $0.736 M_N$  for symmetric nuclear matter. Meanwhile the curves for the equation of states (EOS) are obtained too. The numerical results show that the EOSs for the two sets of parameters are quite close to each other. we display then only the equation of states for the parameter set  $C_2$  in Fig. 1. The figure shows evidently that the symmetric nuclear matter can exist stably. However the stable pure neutron matter does not exist.

It has been shown that the neutral pion condensate may appear in both symmetric nuclear matter and pure neutron matter at densities of  $0.32 \text{ fm}^{-3}$ ,  $0.2 \text{ fm}^{-3}$ , respectively [18, 19]. From the Eq. (23) we obtain  $\langle \vec{\pi}^2 \rangle > 0$  when  $\rho_s > \frac{m_\pi^2 f_\pi^2}{4\delta M_N}$ , where  $\rho_s = \langle \bar{\psi}\psi \rangle$ , is the scalar density of nucleons. Fig. 2 shows the expectation value of  $\sqrt{\langle \vec{\pi}^2 \rangle}$  in nuclear matter as a function of the number density of nucleons  $\rho_N$ . For symmetric nuclear matter  $\langle \vec{\pi}^2 \rangle$  is not zero when  $\rho_N > 0.235 \text{ fm}^{-3}$  which is about 1.5 times the saturation density  $\rho_0$  for the parameters  $C_2$ . Thereafter the pion condensate increases with the increasing of the nuclear matter density. It shows obviously that pion condensate will happen in the nuclear matter in this model. Meanwhile the  $\sqrt{\langle \vec{\pi}^2 \rangle}$  in pure neutron matter are almost equal to its value in the symmetric nuclear matter. It manifests that the SU(2) isospin symmetry of nuclear matter has little effect on pion condensate in nuclear matter. As the density of the matter increases, the value of  $\sqrt{\langle \vec{\pi}^2 \rangle}$  grows rapidly. The pion condensate would play an important role in neutron stars.

Making use of the Hellmann-Feynman theorem and Eq.(17-18), one has the relation between the quark condensate in nuclear matter  $Q(\rho)$  and that in vacuum  $Q(0)$  as

$$\begin{aligned} \frac{Q(\rho)}{Q(0)} &= 1 + \frac{1}{2Q(0)} \left( \sum_{N=n,p} \frac{\partial \varepsilon}{\partial M_N} \frac{dM_N}{dm_q} + \frac{\partial \varepsilon}{\partial M_\sigma} \frac{dM_\sigma}{dm_q} + \frac{\partial \varepsilon}{\partial M_\pi} \frac{dM_\pi}{dm_q} \right) \\ &= 1 - \frac{2\sigma_N}{(m_\pi f_\pi)^2} (\rho_S(p) + \rho_S(n)) - \frac{\pi_0^2}{f_\pi^2} - \frac{3g_3 m_\pi^2 \sigma_0^2}{2m_\sigma^4} \end{aligned} \quad (27)$$

where  $\sigma_N$  is the nucleon  $\sigma$  term.

With the parameter sets  $C_2$  determined above and  $\sigma_N = 45$  MeV, we evaluate the ratio of the in-medium quark condensate to that in vacuum. The obtained results are represented in Fig. 3. From the figure one can easily know that the quark condensate in nuclear matter decreases, in general, as the nuclear matter density increases. It manifests that the “upturn” problem disappears in the present chiral symmetry breaking approach. Investigating the figure more carefully, one may know there exists a density  $\rho = 0.235 \text{ fm}^{-3}$  at which the derivative of the condensate against the density does not continue. In more detail, as the density is larger than  $0.235 \text{ fm}^{-3}$ , the decreasing rate is increased. Such a behavior is consistent with the result given in Ref.[21]. Since the density  $0.235 \text{ fm}^{-3}$  is just that for the pion condensate to emerge, the result indicates that the pion condensate and the underlying chiral symmetry breaking play very important roles in the quark condensate.

In summary, a nonlinear realization of SU(2) chiral symmetry spontaneous breaking approach is developed in the composite operator formalism. In the mean-field approximation, the properties of symmetric nuclear matter and pure neutron matter are investigated. Meanwhile pion condensate and quark condensate are evaluated. It shows that pion condensate can appear in neutron stars, in nuclear matter at a density higher than the normal nuclear matter density. Moreover, the quark condensate decreases monotonously with the increasing of nuclear matter density, then the chiral symmetry is restored gradually in nuclear matter as the density increases.

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Table 1. The parameters in the calculation of the SU(2) chiral symmetry spontaneous breaking model with  $m_\pi = 139.57 MeV$  and  $f_\pi = 130 MeV$  ( $m_i (i = \sigma, \omega, \rho)$ ,  $M_N$  and  $\delta M_N$  in Mev,  $g_2$  in  $fm^{-1}$ ).

	$g_\sigma(N)$	$m_\sigma$	$g_\omega(N)$	$m_\omega$	$g_\rho(N)$	$m_\rho$	$g_2$	$g_3$	$M_N$	$\delta M_N$
$C_1$	9.111	540	10.587	783	8.480	770	-4.0	20.0	888	50
$C_2$	9.111	570	9.573	783	8.480	770	-9.0	37.5	888	50

## Figure Captions

**Fig. 1.** Obtained average energy per nucleon  $\varepsilon/\rho_N - M_N$  as a function of nucleon density  $\rho_N$  for the parameter sets  $C_2$ . The solid line denotes symmetric nuclear matter , and the dashed line is for pure neutron matter .

**Fig. 2.** Obtained expectation value of  $\sqrt{\vec{\pi}^2}$  versus nucleon density  $\rho_N$  in the nuclear matter for the parameter sets  $C_2$ . Same case as in Fig. 1.

**Fig. 3.** Obtained ratio of the quark condensate in nuclear matter to that in vacuum as a function of nucleon density for the parameter sets  $C_2$ . Same case as in Fig. 1.